Effects of the merger history on the merger rate of primordial black hole binaries

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Outline

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Introduction

Up to now, ten binary black holes (BBHs) have been detected by LIGO/Virgo, which implies

- There are many BBHs in our universe;
- BBHs can merge within Hubble time;
- Black holes have mass function.

However, we still do not know

- Where do these BHs come from?
- What is the origin of such heavy BHs?
- What is the formation mechanism for these binaries?

One might suspect that these BHs are of primordial origin, namely, primordial black holes (PBHs). Depending on the model, primordial black holes could have initial masses ranging from $10^{-8}kg$ (Planck relics) to more than thousands of solar masses.

Motivations and origins of primordial black hole

Motivations

- a perfect candidate for dark matter
- to provide seeds for supermassive BHs
- to provide seeds for cosmic structures
- to account for LIGO events

Origins

- early universe phase transitions(*i.e.* vacuum bubbles)
- topological defects(*i.e.* cosmic strings, domain walls)
- from preheating after inflation(*i.e.* preheating, oscillons)
- from large perturbations on small scales produced during inflation
- from the matter-dominated phase of the Universe

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Formation and merger history of PBH binaries

- PBHs decouple from the cosmic background expansion in the radiation era;
- Two PBHs form a binary through Newtonian gravity;
- The surrounding PBHs, especially the nearest PBH, will exert torques on the PBH binary. The tidal force will provide an angular momentum to prevent this system from direct coalescence;
- PBH binaries will merge through gravitational radiation and be detected by LIGO/Virgo.
- A PBH binary merges into a new black hole which together with another PBH form a new PBH binary. Such a second-merge event can in principle be detected by LIGO/Virgo at the present time.

Our work

We aim to work out the merger rate density of PBH binaries by taking into account

- PBH have a general mass distribution;
- Torque from the nearest PBH;
- The merger history of PBH binaries;

Assumptions:

- Spatial distribution of PBHs is random one;
- Negligible initial peculiar velocities and accretion;

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Some definitions

- The probability distribution function of PBHs $P(m)$ is normalized to be

$$\int dm P(m) = 1. \tag{1}$$

- f_{pbh} is the fraction of PBHs in DM.
- The present total average number density of PBHs, n_T, is given by

$$n_T \equiv f_{\mathrm{pbh}} \rho_{\mathrm{dm}} \int dm \frac{P(m)}{m}. \tag{2}$$

- For simplicity, we define m_{pbh} and $F(m)$ as

$$\frac{1}{m_{\mathrm{pbh}}} = \int dm \frac{P(m)}{m}, F(m) \equiv \frac{n(m)}{n_T} = P(m) \frac{m_{\mathrm{pbh}}}{m}. \tag{3}$$

Single-merger events

Torques by nearest PBH

$$\mathscr{R}_1 \quad (t, m_i, m_j) = F(m_i) F(m_j) \int F(m_l) (m_l)^{-\frac{21}{37}} dm_l (m_{\rm pbh})^{-\frac{53}{37}}$$
$$\times \quad 1.32 \times 10^6 (M_\odot)^{\frac{32}{37}} \left(\frac{t}{t_0}\right)^{-\frac{34}{37}} (m_i m_j)^{\frac{3}{37}} (m_i + m_j)^{\frac{36}{37}} f_{\rm pbh}^{\frac{53}{37}} \quad (4)$$

Liu, Guo, Cai. Phys.Rev. D99 (2019)
Torques by all PBHs

$$\mathscr{R}_1 \quad (t, m_i, m_j) = F(m_i) F(m_j) (1 + \frac{\sigma_{\rm eq}^2}{(0.85 * f_{\rm pbh})^2})^{-\frac{21}{74}} m_{\rm pbh}^{-2}$$
$$\times \quad 1.54 \times 10^6 (M_\odot)^{\frac{32}{37}} \left(\frac{t}{t_0}\right)^{-\frac{34}{37}} (m_i m_j)^{\frac{3}{37}} (m_i + m_j)^{\frac{36}{37}} f_{\rm pbh}^{\frac{53}{37}}. \quad (5)$$

Multiple-merger events

The merger rate density of PBH binaries with the masses m_i and m_j in the second-merger process is given by

$$\begin{aligned}\mathscr{R}_2 \quad & (t, m_i, m_j) = \frac{1}{2}\int dm_l dm_e \mathscr{R}_2(t, m_i - m_e, m_e, m_j, m_l) \\ + \quad & \frac{1}{2}\int dm_l dm_e \mathscr{R}_2(t, m_j - m_e, m_e, m_i, m_l). \qquad (6)\end{aligned}$$

where

$$\begin{aligned}\mathscr{R}_2 \quad & (t, m_i, m_j, m_k, m_l) = F(m_i) F(m_j) F(m_k) F(m_l) \\ \times \quad & 1.59 \times 10^4 (M_\odot)^{\frac{27}{37}} \left(\frac{t}{t_0}\right)^{-\frac{31}{37}} (m_i + m_j)^{\frac{6}{37}} (m_k)^{\frac{6}{37}} \\ \times \quad & (m_l)^{-\frac{42}{37}} (m_{\rm pbh})^{-\frac{69}{37}} (m_i + m_j + m_k)^{\frac{72}{37}} f_{\rm pbh}^{\frac{69}{37}}. \qquad (7)\end{aligned}$$

- The total merger rate density of PBH binaries with the masses m_i and m_j detected by LIGO-Virgo is given by

$$\mathscr{R}(t, m_i, m_j) = \sum_{n=1} \mathscr{R}_n(t, m_i, m_j)\,. \tag{8}$$

- The merger rate of PBH binaries in in n-th merger process at the time t is given by

$$R_n(t) \equiv \int\int \mathscr{R}_n(t, m_i, m_j) dm_i dm_j. \tag{9}$$

- In the single-merger case, we have $\alpha = -(m_i + m_j)^2 \partial^2 \ln \mathscr{R}(t, m_i, m_j)/\partial m_i \partial m_j = 36/37$ which is independent of the PBH mass function. However, by taking account into the merger history of PBHs, α depends on the PBH mass function, which could help us reconstruct the mass function of PBHs.

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Monochromatic case

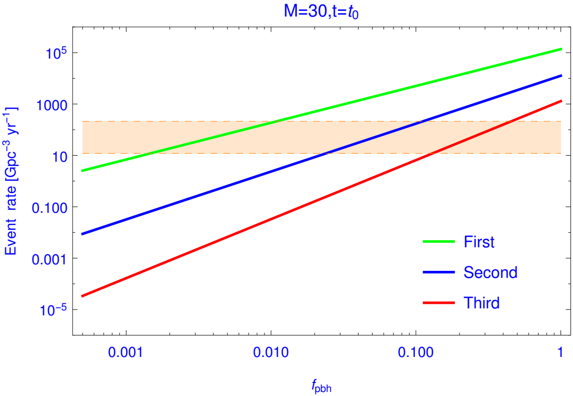

Figure: Event rate of first-merger (green), second-merger (blue) and third-merger (red) of PBH binaries with the mass $30M_\odot$ at the present time as a function of the PBH abundance.

Power-law case

- In this subsection, we take the PBH mass function as a power-law form:

$$P(m) \approx \frac{q-1}{M}\left(\frac{m}{M}\right)^{-q}. \tag{10}$$

In the power-law case, we can arrive:

$$m_{\mathrm{pbh}} = M\frac{q}{q-1}, \tag{11}$$

$$F(m) = \frac{q}{m}\left(\frac{m}{M}\right)^{-q} \tag{12}$$

Power-law case

- Choosing $f_{\rm pbh} = 0.01$, $q = 2.3$, $M = 0.2M_\odot$, we can get $R_1(t_0) = 9.66 \times 10^3$ Gpc^{-3} yr^{-1}, $R_2(t_0) = 1.15 \times 10^2$ Gpc^{-3} yr^{-1}, $R_3(t_0) = 5.00$ Gpc^{-3} yr^{-1}.
- In power-law case, the effect of the merger history on the merger rate of PBH binaries is small. However, the effect of the merger history on the merger rate density is significant in some region of the parameter space.
- For example, $\mathscr{R}_1(t_0, 30M_\odot, 30M_\odot) = 8.55 \times 10^{-7}$Gpc^{-3} yr$^{-1}M_\odot^{-2}$, $\mathscr{R}_2(t_0, 30M_\odot, 30M_\odot) = 8.90 \times 10^{-7}$Gpc^{-3} yr$^{-1}M_\odot^{-2}$, $\mathscr{R}_3(t_0, 30M_\odot, 30M_\odot) = 4.88 \times 10^{-8}$Gpc^{-3} yr$^{-1}M_\odot^{-2}$.

Power-law case

- There are several gravitational wave events detected by LIGO/Virgo. Masses of black hole all are in $(5M_\odot, 50M_\odot)$. In such region, in the future, more and more coalescence events of black hole binaries will be detected by LIGO/Virgo. When we use the merger rate distribution to fit the mass function of PBH, the effect of merger history on the merger rate density of PBH binaries can not be ignored.

Two typical mass function

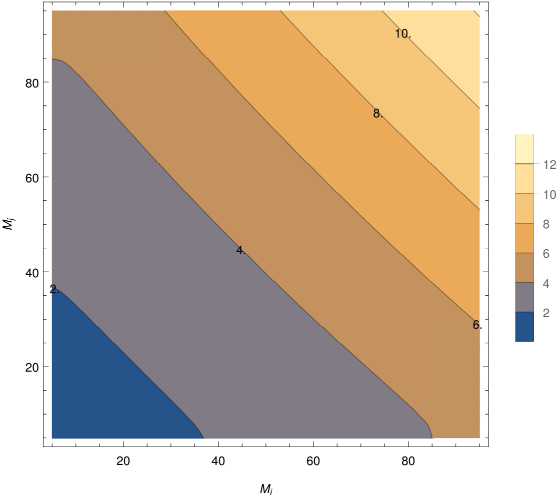

Figure: Contour of the ratio of the total merger rate density to the single-merger one in the PBH mass plane in the case of $f_{\rm pbh} = 0.01$, $q = 2.3$ and $M = 0.2 M_\odot$.

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Conclutions

- We develop a formalism to calculate the merger rate density of primordial black hole binaries with a general mass function, by taking into account the merger history of primordial black holes.
- We apply the formalism to two specific mass functions, monochromatic and power-law cases. In the former case, the merger rate is dominated by the single-merger events, while in the latter case, the contribution of the multiple-merger events on the merger rate density can not be ignored.
- The effects of the merger history on the merger rate density depend on the mass function.